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USING THE APPROXIMATION OF A GENERALIZED TELEGRAPHIC PROCESS FOR CALCULATION OF THERMAL RADIATION OF TURBULENT DIFFUSION FLAMES

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A new method based on stochastic modeling with the use of the approximation of the generalized telegraphic process is suggested to calculate thermal radiation of turbulent high-temperature flows. The computational advantages of this method are discussed in comparison with traditional approaches to stochastic modeling. The considerable effect of fluctuations of temperature and concentrations of radiating components on the formation of fields of brightness and intensity of radiation of a turbulent diffusion flame of a mixture of hydrogen and carbon oxide in air is demonstrated. The results of numerical simulation are compared with experimental data.

Transfer of radiation in nonisothermal flows of molecular gases is an important constituent of such problems as radiative heat transfer, optical diagnostics, and prediction of the state of the atmosphere and climate. Since most natural and technogenic flows — jets, flames, thermals, wind flows, etc. — are turbulent, there exists a problem of adequate account for the effect of fluctuations of temperature and concentrations on the field of thermal radiation. One of the main problems is the determination of the mean (time- or ensemble-averaged) intensity of radiation, which, due to the nonlinear dependence of optical characteristics of a medium on temperature, can differ noticeably from the corresponding value calculated from mean thermodynamic parameters.

The importance of correct account for the effect of turbulent fluctuations on thermal radiation in solving problems of optical diagnostics of the jets of combustion products of hydrocarbon fuels in the atmosphere was shown in [1–3]. Actually, the main radiation in the jets mentioned in the IR range of the spectrum is caused by H_2O and CO_2 molecules; these very molecules absorb IR radiation in the atmosphere. Consequently, radiation from a jet passes through the bulk of the atmosphere and arrives at a receiver mainly in spectral ranges with a rather sharp dependence of the coefficient of absorption on temperature in those places where the radiating capacity of the hot jet, which is proportional to the coefficient of absorption, is high, while absorption in the cold atmosphere is poor.

The sharp dependence of optical characteristics on temperature presupposes the substantial influence exerted on radiation by temperature fluctuations, whose contribution must be allowed for in calculations. This allowance requires averaging of the equation of radiation transfer:

$$\frac{d\,\langle I_\nu \rangle}{dx} = \langle k_\nu B \rangle - \langle k_\nu I_\nu \rangle \,. \tag{1}$$

With such an approach, the main difficulties arise in determining of the last term on the right-hand side of Eq. (1). Since the intensity of radiation is determined by temperature and concentration at all the points on

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the beam, the operation of averaging requires knowledge of the multidimensional probability density function. The situation may become much simpler when the path length of a quantum in a radiating medium is much larger than the outer scale of turbulence Λ:

$$\frac{1}{k_\nu} >> \Lambda \, . \tag{2}$$

In this case, the mean product $\langle k_\nu I_\nu \rangle$ is split into a product of mean co-factors and the allowance for turbulent fluctuations is reduced to local averaging of the coefficients in the equation of radiation transfer:

$$\frac{d \langle I_\nu \rangle}{dx} = \langle k_\nu B \rangle - \langle k_\nu \rangle \langle I_\nu \rangle \, . \tag{3}$$

This approach has been termed the approximation of optically thin fluctuations [4]. Its use in the discussed problems of optical diagnostics is justified because the radiation for which condition (2) is violated is, as a rule, "filtered" by periphery regions of the jet or by the atmosphere.

In the heat-exchange systems based on the use of nonreacting high-temperature jets ($T > 1000$ K), the contribution of turbulent fluctuations to radiant energy flux to the wall turns out to be insignificant. According to estimates [4] and the results of numerical studies [5], this contribution does not exceed several percent relative to the quantities calculated from the mean thermodynamic parameters, which is comparable with the error of the methods used for calculations. The insufficient influence of fluctuations on radiation is attributable to the small temperature gradients in these systems and, consequently, the small amplitudes of temperature fluctuations. Moreover, the contribution of spectral ranges with a sharp temperature dependence of optical characteristics to total radiant energy fluxes is not determining.

However, in heat-exchange systems with combustion, the amplitudes of fluctuations of temperature and concentrations may be appreciable, accounting for the substantial effect of turbulent fluctuations on thermal radiation transfer. Interaction of turbulence and radiation in these systems was studied, for example, in [6–8]. However, despite the great efforts made to solve this problem, many of its aspects have not yet been fully clarified. The quantitative estimates of the contribution of fluctuations obtained in various works are also rather contradictory. The objective of the present study is the choice of a method for calculating the thermal radiation of high-temperature turbulent flows which would be free from the limitations on the amplitude of the fluctuations of temperature and concentrations, and the use of this method to investigate the processes of radiative transfer in turbulent diffusion flames.

For the analysis of the effect of turbulence on radiation at arbitrary amplitudes of fluctuations, it was suggested in [9] to use the approximation of a generalized telegraphic process (GTP). However, the result obtained there was suitable only for a statistically uniform layer, which precludes the study of actual turbulent jets and flames. In the present paper, the approximation of a generalized telegraphic process, implemented numerically within the framework of stochastic modeling, is used to calculate radiation of inhomogeneous media.

The methods of stochastic modeling allow one to obtain instantaneous values of fluctuating quantities (temperature, concentrations, etc.) along a certain path. In modeling, information on local probability density functions at all the points of the path and also the coefficients of space and time correlation are used. The thus-obtained instantaneous profiles of thermodynamic parameters are used to calculate instantaneous values of the spectral density of energy brightness (SDEB) I_ν that make it possible to calculate the required moments of the distribution of I_ν, viz., mean values, dispersion, etc.

The methods of stochastic modeling for the calculation of radiation of turbulent flows are used rather extensively [7, 10]. At the present time, the most widely used approach is modeling of instantaneous realiza-

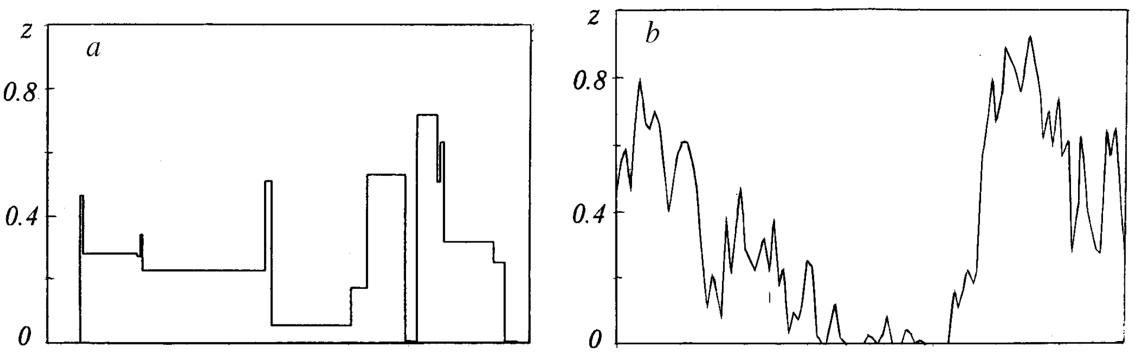

Fig. 1. Instantaneous realizations of the concentration of a passive admixture along a certain path obtained by the approximations of GTP (a) and the process of SRSM (b).

tion of a fluctuating quantity on the basis of a mixed process of self-regression of first order in time and a sliding mean in space (SRSM) [7]:

$$z'_k(t) = R_k(\Delta t)\, z'_k(t - \Delta t) + \sum_{i=1}^{k} \phi_{ik}\, \psi_i(t)\,. \tag{4}$$

Here $z'_k(t)$ is the instantaneous value of the fluctuating quantity z' at the kth point of the path at the time instant t, ϕ_{ik} are the weighting factors necessary to ensure the prescribed space correlation along the path, and $\psi_i(t)$ are the noncorrelated random forces with a zero mean and prescribed dispersion which is chosen so that the required form of the probability density function for z' would be ensured.

In using the approximation of the generalized telegraphic process, an instantaneous stepwise profile of a fluctuating quantity is generated. In this case, the modeling involves the following procedures:

1. Drawing of the number of path partitions (the partition segments qualitatively model separate turbulent vortices in the path). Within the framework of the generalized telegraphic process, the number of partitions is distributed according to the Poisson law with the prescribed length of the path and the length of temperature correlation.

2. Modeling of the position of the points of path partition distributed with uniform density and determination of the sizes of the partitions with constant values of thermodynamic characteristics ("vortices").

3. Modeling of instantaneous realizations of a fluctuating quantity on each segment using random quantities with the prescribed probability density function.

Random quantities drawn within the framework of the generalized telegraphic process have an exponentially decreasing space correlation [11]. According to experimental data, the same correlation function is observed in turbulent jet flows.

Figure 1 presents examples of the concentration profiles of a passive admixture z obtained using the approximations of the generalized telegraphic process (a) and the process of self-regression of first order in time and a sliding mean in space (b). It is seen that the generalized telegraphic process has certain computational advantages, since it admits piecewise-analytical solution of the equation of radiation transfer (1), thus allowing one to obviate standard difficulties in integration of this equation along a strongly nonisothermal path:

$$I_\nu = \sum_{l=1}^{N} B_l \left[1 - \exp(k_l s_l)\right] \prod_{i=0}^{l-1} \exp(-k_i s_i)\,. \tag{5}$$

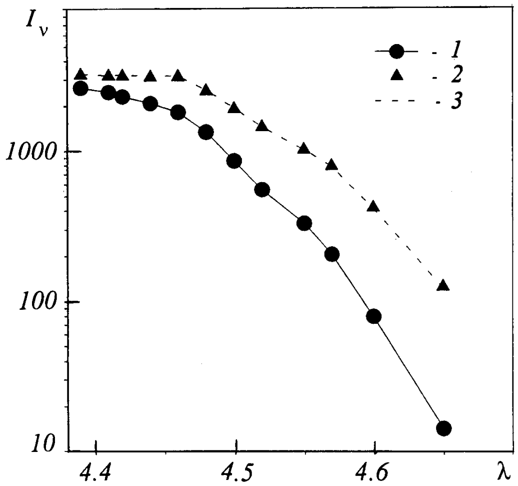

Fig. 2. Spectral density of energy brightness I_ν ($W \cdot m^{-2} \cdot \mu m^{-1} \cdot sr^{-1}$) of a statistically uniform layer: 1) calculation from mean thermodynamic parameters; 2 and 3) calculation on the basis of stochastic modeling using the approximations of GTP (2) and the process of SRSM (3). λ, μm.

Here B_l and k_l are the values of the Planck function and of the coefficient of absorption within the lth step, respectively, s_l is the size of the lth step, $k_0 = 0$, and $s_0 = 0$.

On the other hand, it is obvious that the stepwise profile presented in Fig. 1a is a rather rough approximation of actual distributions of fluctuating quantities in turbulent flows. This raises the question concerning the adequacy of the use of these profiles in calculation of thermal radiation. To answer this question, we compared mean values of the spectral density of energy brightness calculated by stochastic modeling with the use of the approximations of the generalized telegraphic process and the process of self-regression of first order in time and a sliding mean in space. In the calculations we used the same probability density functions of temperature and concentrations and the same lengths of correlations (within the framework of the process of self-regression of first order in time and a sliding mean in space, we took an exponentially decreasing function of space correlation which is realized automatically with the use of the generalized telegraphic process). Noticeable differences (10% or more) in the results of the calculations were obtained for model problems in which very sharp temperature dependences of the coefficient of absorption and of the Planck function were taken. In this case, mean values of the spectral density of energy brightness calculated within the framework of the generalized telegraphic process systematically exceed similar quantities obtained on the basis of the process of self-regression of first order in time and a sliding mean in space. Under the conditions of actual flows studied in the present paper (range of temperature variation 300–2500 K, spectral range 2–6 μm, radiating components CO_2, H_2O, and CO), rather moderate temperature dependences of the optical parameters are realized, which are attributable to slight (not exceeding several percent) differences in the mean values of the spectral density of energy brightness calculated by the two methods mentioned.

Figure 2 presents the spectral density of energy brightness in the wing of the 4.3-μm CO_2 band for a statistically uniform layer with the following parameters: layer thickness $L = 1$ m, correlation length $\Lambda = 0.1$ m, mean temperature $T_0 = 1000$ K, and mean volumetric concentration of CO_2 $f_0 = 0.01$. As the probability density function of temperature and concentrations, we used the "truncated" normal distribution with standard deviations amounting to 20% of the corresponding mean values. As minimum admissible values in drawing of instantaneous profiles, we used $T_{\min} = 300$ K and $f_{\min} = 0$. It is seen from Fig. 2 that against the background of substantial contribution of fluctuations to radiation, the discrepancies between the results obtained on the basis of two different methods of stochastic modeling are insignificant. Similar results of comparison were obtained for nonuniform flows. The described methods of stochastic modeling were used in numerical

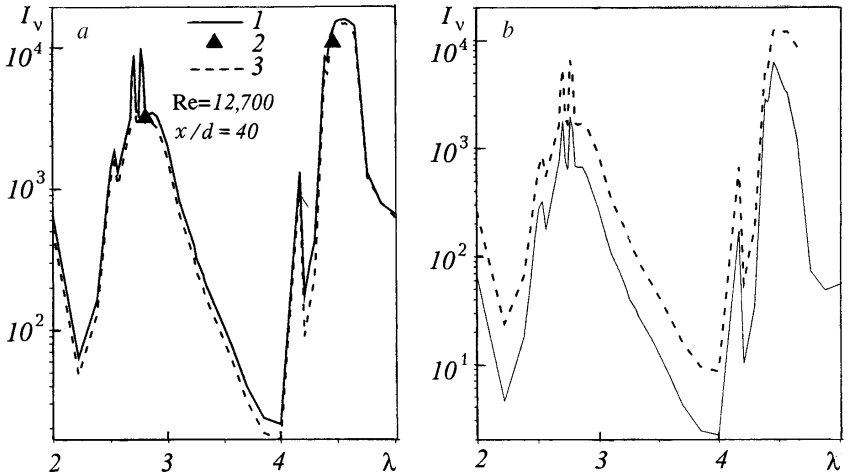

Fig. 3. SDEB I_ν of a turbulent diffusion flame of H_2/CO/air for the beam passing through the flame axis (a) and the beam distant from the flame axis by $\Delta = 3d$ (b): 1) calculation from mean thermodynamic parameters; 2) experimental data of [7]; 3) calculation in the approximation of GTP.

studies of the effect of fluctuations of temperature and concentrations on thermal radiation of turbulent diffusion flames.

The flames mentioned, which are also called unmixed flames, are formed during the mixing of initially separated flows of a fuel and oxidizer. Further, we consider these flows in which the time scale of chemical reactions τ_c is much smaller than the time scale of molecular mixing τ_m (i.e., the Damköler number $\mathrm{Da} = \tau_c/\tau_m >> 1$). In this case, the thermochemical state of the reacting mixture is entirely determined by the concentration of the passive admixture z [12]:

$$Y_i = Y_i^{\mathrm{e}}(z) , \quad T_i = T_i^{\mathrm{e}}(z) . \tag{6}$$

The results presented in what follows have been obtained for axisymmetric turbulent diffusion flames of an H_2/CO mixture in air; detailed experimental studies of these flames were conducted in [7]. By analyzing the laser signal scattered on a disperse additive we recovered instantaneous distributions of the concentration of the passive admixture and on their basis determined the statistical characteristics necessary for stochastic modeling (local probability density functions, lengths of space correlations, etc.). In [7], the equations of state (6) are also given for the studied flame; they make it possible to recover the instantaneous values of temperature and concentrations of radiating components (in our case CO_2, H_2O, and CO) on the basis of instantaneous values of z. Vast samplings of the instantaneous values of the spectral density of energy brightness for the beams passing through the flame axis for three distances from the initial section – 30d, 40d, and 50d – were obtained experimentally for two values of wavelengths in the IR range. Different statistical characteristics of radiation (mean values, dispersions, spectra of intensity, etc.) which can be used for comparison with the results of numerical modeling were calculated by processing the instantaneous values of the density spectra of energy brightness. The measurements mentioned were made for two modes of outflow for which there correspond the Reynolds numbers Re = 7400 and 12,700 in the initial cross section of the flow.

Figure 3 presents the values of the spectral densities of energy brightness calculated from mean thermodynamic parameters and by the method of stochastic modeling in the approximation of the generalized telegraphic process for the spectral range of 2–5 μm and also the measured values of the spectral density of energy brightness for two wavelengths. The spectral densities of energy brightness were calculated by the

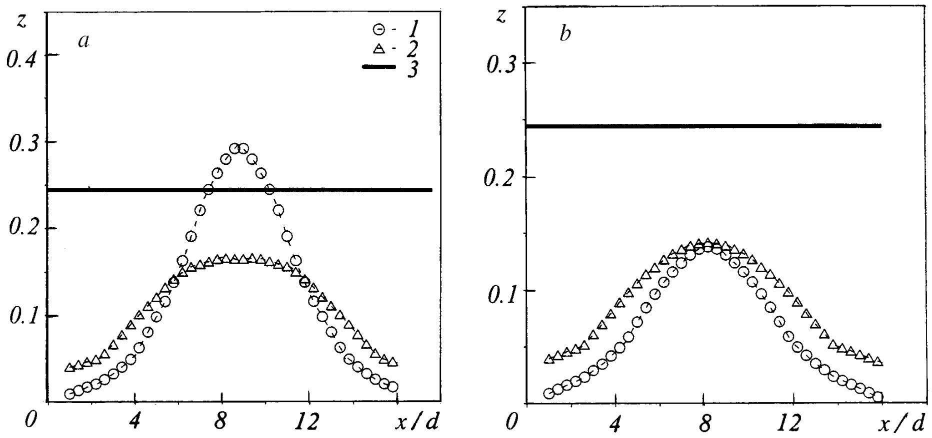

Fig. 4. Distribution of statistical characteristics of concentration of a passive admixture along the beam passing through the flame axis (a) and the beam distant from the flame axis by $\Delta = 3d$ (b): 1) mean values; 2) root-mean deviations; 3) stoichiometric value of z.

Curtis–Godson method [13]. It is seen that for the beam passing through the flame axis (Fig. 3a) the contribution of fluctuations changes the value of radiative flux weakly compared to the results of calculations from mean temperatures and concentrations (the differences do not exceed 15–20%). In this case, calculations both from mean parameters and on the basis of stochastic modeling lead to a satisfactory agreement with experimental data. The results of comparison for other situations, for which the measurement data are given in [7], are similar: allowance for turbulent fluctuations leads to a 10–20% change in the mean intensity of radiation compared to the quantities calculated from mean parameters, with the contribution of fluctuations changing the value of radiative flux to either side. Here, in all cases there is a satisfactory agreement between calculations and measurements.

The fact that turbulent fluctuations in a diffusion flame can lead to both increase and decrease in mean radiation emitted from the flame can be explained by analyzing the equation of state for temperature. The dependence of T on z has a maximum at a stoichiometric value of the concentration of the passive admixture z_s, equal to 0.244 for the conditions considered [7]. If the mean value of z at some point of the flame is close to z_s, then fluctuations of z to both greater and smaller values lead to a decrease in temperature, and, consequently, to a decrease in the emerging thermal radiative flux (according to the experimental data of [7], z has a truncated normal distribution).

Figure 4a shows distributions of mean values and root-mean deviations of z along the path for which the radiation spectrum presented in Fig. 3a was calculated. It seen that the path has sections where $\bar{z}$ is close to z_s — here the contribution of fluctuations leads to a decrease in the emerging radiative flux. In the sections where $\bar{z}$ is noticeably smaller than z_s, the effect of fluctuations usually leads to an increase in the emerging radiative flux. For the path under consideration, the effects mentioned compensate each other to a great extent and the resultant effect of turbulent fluctuations on the mean spectral density of energy brightness turns out to be slight. This kind of situation occurs for all paths where the spectral density of energy brightness was measured [7]. Therefore, these data are likely to be suitable for verifying the adequacy of the methods of allowance for the effect of fluctuations on radiation transfer (although in some studies, e.g., [10], the data of [7] are used just for these purposes).

One can expect a more considerable effect of fluctuations of temperature and concentrations on the radiation of a turbulent diffusion flame for the paths along which $\bar{z} < z_s$, for example, for the beams intersecting the cross sections of the considered flames in the chord, which is at a distance Δ from the flame axis, rather than in the diameter. Figure 4b presents the distributions of $\bar{z}$ and σ_z along the beam corresponding to

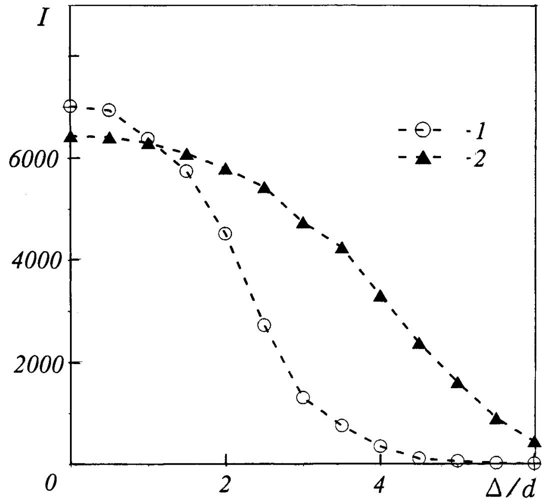

Fig. 5. Dependence of SDEB I ($W \cdot m^{-2} \cdot sr^{-1}$) integrated over the spectral range of 2–5 μm on the distance along the line of sight from the flame axis: 1) calculation from mean thermodynamic parameters; 2) calculation in the approximation of the GTP.

$\Delta = 3d$. The calculation results for the spectral density of energy brightness for this beam are shown in Fig. 3b. The contribution of turbulent fluctuations to a mean radiative flux in this case can noticeably exceed the values of the spectral density of energy brightness calculated from mean thermodynamic parameters. Figure 5 presents the dependence of the values of the spectral density of energy brightness integrated over the spectral range of 2–5 μm on Δ. It is seen that the contribution of fluctuations greatly changes the field of flame brightness and noticeably increases the intensity of radiation from the total cross section of the flame: for the cross section 40*d*-distant from the flame origin, the difference of the spectrum-integral radiation intensity, calculated by stochastic modeling, from the corresponding quantity determined from mean parameters is about 50%. Similar results were obtained for other cross sections of turbulent diffusion flames of H_2/CO in air, for which the data needed to calculate radiation is published in [7].

The results obtained in the present paper indicate that turbulent fluctuations of temperature and concentrations can make a considerable contribution to the formation of thermal radiation of turbulent diffusion flames and they must adequately be allowed for in solving problems of radiative heat transfer. The methods of stochastic modeling, including the suggested approach based on the use of the approximation of the generalized telegraphic process, may be used effectively for quantitative description of the contribution of turbulent fluctuations to radiation.

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NOTATION

I_ν, spectral density of energy brightness; k_ν, coefficient of absorption; B, Planck function; x, coordinate reckoned along the line of sight; Λ, outer scale of turbulence (correlation length); z, mass concentration of a passive admixture; $\bar{z}$, mean value of z; z', fluctuating part of z; σ_z, root-mean deviation of z; T, temperature; f, volumetric concentration; t, time; R, coefficient of time correlation; d, diameter of the initial section of flame; Δ, distance from the line of sight to the flame axis; Y_i, mass concentration of the ith component; λ, wavelength; τ, time scale of the process; $\langle ... \rangle$, averaging over turbulent fluctuations. Subscripts: ν, frequency; e, equilibrium; c, chemical; m, mixing; 0, initial value.

REFERENCES

1. Yu. V. Belyaev, A. I. Bril', O. B. Zhdanovich, and Yu. V. Khodyko, *Fiz. Goreniya. Vzryva*, No. 6, 92–97 (1990).
2. V. P. Kabashnikov, N. V. Kuz'mina, A. A. Kurskov, and G. I. Myasnikova, *Teplofiz. Vys. Temp.*, **31**, No. 6, 962–966 (1993).
3. A. Bril', V. Kabashnikov, Yu. Khodyko, and O. Zhdanovich, *Proc. SPIE*, **3110**, 68–72 (1997).
4. V. M. Ievlev, *Turbulent Motion of High-Temperature Continua* [in Russian], Moscow (1975).
5. S. Mazumder and M. F. Modest, *ASME J. Heat Transfer*, **121**, 726–729 (1999).
6. T. H. Song and R. Viskanta, *Aérokosmich. Tekh.* [in Russian], No. 7, 86–94 (1987).
7. M. E. Kounalakis, J. P. Gore, and G. M. Faeth, *J. Heat Transfer*, **111**, 1021–1030 (1989).
8. S. Mazumder and M. F. Modest, *Int. J. Heat Mass Transfer*, **42**, 971–991 (1998).
9. V. P. Kabashnikov, *Energy*, **23**, No. 2, 113–123 (1998).
10. S. H. Chan, X. C. Pan, and J. Zhang, in: *Proc. 25th Int. Symp. on Combustion*, The Combustion Institute, Pittsburgh (1994), pp. 1115–1123.
11. V. I. Klyatskin, *Stochastic Equations and Waves in Randomly Inhomogeneous Media* [in Russian], Moscow (1980).
12. R. W. Bilger, *Ann. Rev. Fluid Mech.*, **21**, 101-135 (1987).
13. C. B. Ludwig, W. Malkmus, J. E. Reardon, and J. A. L. Thompson, *Handbook of Infrared Radiation from Combustion Gases,* NASA SP-3080, Huntsville, Ala (1973).